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Advanced Statistical Physics - Problem set 10

Summer Terms 2022

Hand in: Hand in tasks marked with * to mailbox no. (43) inside ITP room 105b by Friday 17.06. at 9:15 am.

16. Upper critical dimension and Ginzburg criterion* *2+2+1 Points*

Consider the Ginzburg-Landau Hamiltonian

$$\mathcal{H} = \int d^d x \left[\frac{a\tau}{2}\psi^2 + \frac{c}{2}(\nabla\psi)^2 + \frac{u}{4}\psi^4 - h\psi \right],$$

where $a, c, u > 0$ and $\tau = (T - T_c)/T_c$ is the reduced temperature.

a) Rescale the variables

$$\psi(\boldsymbol{x}) = \psi_0 \psi'(\boldsymbol{x}), \quad h(\boldsymbol{x}) = h_0 h'(\boldsymbol{x}) \text{ and } \boldsymbol{x} = x_0 \boldsymbol{x}'$$

and show that by choosing ψ_0, h_0 and x_0 properly, the Ginzburg-Landau Hamiltonian can be written as

$$\frac{\mathcal{H}}{T} = \left(\frac{|\tau|}{\tau_G}\right)^\alpha \int d^d x' \left[\pm \frac{1}{2}\psi'^2 + \frac{1}{2}(\nabla'\psi')^2 + \frac{1}{4}\psi'^4 - h'\psi' \right],$$

where in the first term the $\pm$-sign is determined by the sign of τ. What are the expressions for α and τ_G?

b) Consider the prefactor in the rescaled Ginzburg-Landau Hamiltonian. Under what condition does the saddle point approximation become asymptotically exact in the vicinity of the critical temperature $|\tau| \to 0$? What does this tell you about the upper critical dimension?

c) In the lectures the concept of Ginzburg criterion was defined. What does the above analysis tell you about the Ginzburg reduced temperature?

17. Correlation function II *3+3+3 Points*

Consider the Ginzburg-Landau functional

$$\mathcal{H} = \int d^d x \left[\frac{a\tau}{2}\psi(\mathbf{x})^2 + \frac{c}{2}(\nabla\psi(\mathbf{x}))^2 - h(\mathbf{x})\psi(\mathbf{x}) \right].$$

The associated Euler-Lagrange equation is given by

$$c\nabla^2\psi(\mathbf{x}) = a\tau\psi(\mathbf{x}) - h(\mathbf{x}).$$

a) Use the Fourier transformation to write down the formal solution of this equation for $h(\mathbf{x}) = h\delta^{(d)}(\mathbf{x})$. In the lectures it will be shown that this solution is equivalent with a two point correlation function.

b) Solve the Euler-Lagrange equation for $\tau = 0$ and $h(\mathbf{x}) = h\delta^{(d)}(\mathbf{x})$.
Hint: Use Gauss's theorem.

c) Solve the Euler-Lagrange equation for $\tau > 0$.
Hint: Assume that the solution is spherically symmetric and decays exponentially at large distances

$$\psi(\mathbf{x}) \propto \frac{e^{-r/\xi}}{r^p}.$$

Solve the equation in the limits $r \ll \xi$ and $r \gg \xi$.